BUSINESS MATHEMATICS

(For Undergraduate Classes)

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PREFACE

This Text Book "**Business Mathematics**" is specially prepared for undergraduate Classes. It is prepared in accordance with latest syllabus of the university. Needless to say, only my experience in teaching Business Statistics and Mathematics for B.Com. and BBM for some years, prompted me to do this attempt. I do not claim any orginality in the subject matter of this book. I have drawn immense material from many standard Books on this subject. Yet this book is original in the exposition and presentation of the subject matter.

I thank my family members specially my husband and children for their whole hearted support and encouragement.

I sincerely thank United Publishers,Mangalore for bringing out this book and their co-operation and support.

I shall consider my efforts amply rewarded if this book is found useful to the teachers and students.

Suggestions for further improvement for the same solicited.

Dr. Sudha N. Vaidya

Mangalore

Syllabus

Unit 1 :Formulation of Simple Simultaneous Equation and their Solution 12 hrs.

Quadratic - equations- solution of quadratic equation by factors and by formula; Matrices- Type Basic Concepts addition- substaction - multiplication; Determinants - value of determinant - solving simultaneous equation (with two and three variables) by Cramer's rule.

Unit 2 : Minors and Cofactors of Matrices : 12 hrs.

Adjoint of a matrix, inverse of a matrix - solving simultaneous equations (with two and three variables) by matrix inverse method; Progression- Arithmetic Progression - definition- n^{th} term - sum of n terms - three numbers in AP; Geometric Progression- Definition - n^{th} term - sum of a n terms three numbers in GP - Practical problems related to AP and GP.

Unit 3: Ratio : 12 hrs.

Proportion - Direct Proportion- inverse proportion; Simple interest - payment by installment -True discount Bankers discount and Bankers gain - equated due date; Trade discount- cash discount - invoice price and selling price.

Unit 4 : Logarithms 12 hrs.

Definition - Laws of Logarithms (without proof) common logarithms - antilogarithms - mathematical calculations using common logarithms; compound interest - Formula for compound interest- interest payable half yearly or quarterly - interest for fraction of a period, nominal and effective rates; Annuities - Amount of an immediate annuity and annuity due, present value of annuity immediate and annuity due present values of perpetuity due and perpetuity immediate

Total Hours: 48

CONTENTS

UNIT I FORMULATION OF SIMPLE SIMULTANEOUS EQUATION AND THEIR SOLUTIONS

Formation of simple simultaneous equations and solutions. Quadratic equations, solving quadratic equation by factorisation, by formula matrices. Types basic concepts addition, substraction, multiplication determinants, solving, simultaneous equation two and three variables by Cramers rule.

Quadratic Equation

An equation which when reduced to rational integral form contains the square of unknown quantity and no higher power is called a quardratic equation or an equation of the second degree. An equation which contain only square of unknown and not the first power is called (a) pure quadratic equation. e.g., $5x^2 - 21$

(b) If an equation which contains the square as well as the first power of unknown is called complete quadratic equation.

Eg., $3x^2 - 5x = 0$ expressed as $ax^2 + bx + c_1$ a,b,c are in real no. and a is not equal to zero. This is because a $= 0$ then the expression ax^2 becomes equal to zero and the equation become linear.

(c) **Radical Quadratic Equation :** An equation that contains the surds like $\sqrt{3x-2}$, $\sqrt{ax+b}$, $\sqrt{ax+bx+c}$, $\sqrt[3]{ax+b}$, is called Radical Quadratic Equation. Sometimes such an equation is not obvious from its expression itself. In such a case, the equation is reduced to Quadratic form by the process of squaring.

For Example :

(a) If $\sqrt{3x-5} = x - 3$

Then by squaring both the sides we get $\left(\sqrt{3x-5}\right)^2 = (x-3)^2$

$$3x - 5 = x^2 - 6x + 9$$

$$= \quad x^2 - 6x - 3x + 9 + 5 = 0$$

$$x^2 - 9x + 14x = 0 \text{ is a Quadratic equation}$$

(b) If $\sqrt{3x-2} + \sqrt{x} = 2$

Then $\sqrt{3x-2} = 2 - \sqrt{x}$

Solving the Linear Equations :

Simplify the following equation and reduces into standard form of $ax + b = 0$ were $a \neq 0$. The solution given by $x = \dfrac{-b}{a}$ and $a \neq 0$.

Problem No. 1. $3(x+5) - 25 = 9 + 2(x-7)$ Find the value of x

$$3x + 15 - 25 = 9 + 2x - 14$$

$$3x - 2x = 9 - 14 - 15 + 25$$

$$\mathbf{x = 25}$$

Problem No. 2. $3(4x+1) - (4x-1) = 6(x+10)$

$$12x + 3 - 4x + 1 = 6x + 60$$
$$12x - 4x + 6x = 60 - 3 + 1$$
$$2x = 56$$
$$x = \frac{56}{2}$$
$$\mathbf{x = 28}$$

Problem No. 3. Solve the equation $\frac{1}{x-3} + \frac{1}{x-2} = \frac{1}{x-4} + \frac{1}{x-1}$

$$\frac{1}{x-3} + \frac{1}{x-2} = \frac{1}{x-4} + \frac{1}{x-1}$$
$$\frac{(x-2)+(x-3)}{(x-3)(x-2)} = \frac{(x-1)+(x-4)}{(x-4)(x-1)}$$
$$\frac{2x-5}{x^2-3x-2x+6} = \frac{2x-5}{x^2-4x-x+4}$$
$$\frac{2x-5}{x^2-5x+6} = \frac{2x-5}{x^2-5x+6} \text{ By cross multiplication}$$
$$(2x+5)(x^2-5x+4) = (2x-5)(x^2-5x+6)$$
$$2x^3 - 5x^2 - 10x^2 + 25x + 8x - 20 = 2x^3 - 5x^2 - 10x^2 + 25x + 12x - 30 = 0$$
$$\cancel{2x^3} - \cancel{15x^2} + 33x - 20 - \cancel{2x^3} + \cancel{15x^2} - 37x + 30 = 0$$
$$33x - 37x - 20 + 30 = 0$$
$$-4x + 10 = 0$$
$$-4x = -10$$
$$x = \frac{-10}{4}$$
$$x = \frac{5}{2}$$
$$\mathbf{x = 2.5}$$

Problem No. 4. $\frac{1}{x-1} + \frac{3}{x+4} = \frac{4}{x+3}$

$$\frac{(x+4)+3x+3}{(x+1)(x+4)} = \frac{4}{x+3}$$
$$\frac{4x+7}{(x+1)(x+4)} = \frac{4}{x+3} \text{ By Cross Multiplication}$$
$$(4x+7)(x+3) = (x+1)(x+4)4$$
$$4x^2 + 7x + 12x + 21 = (x^2 + x + 4x + 4)4$$
$$4x^2 + 19x + 21 = (x^2 + 5x + 4)4$$
$$4x^2 + 19x + 21 = 4x^2 + 20x + 16$$
$$4x^2 + 19x + 21 - 4x^2 - 20x - 16 = 0$$
$$-x + 5 = 0$$
$$5 = x$$
$$\mathbf{x = 5}$$

Roots :

The values of x is satisfy the equation $ax^2 + bx + c = 0$ and a is not equal to zero are called roots or solution of the equation. A quadratic equation has 2 roots. The roots of the equation are obtained by means of two methods. The (1) Factorisation Method and (2) By Using Formula.

1. Factorisation Method : In this method, we factorised the quadratic equation $ax^2 + bx + c$ into linear factors.

We $ax^2 + bx + c = (Px + q)(rx + s)$

then $ax^2 + bx + c = 0$

$(Px + q)(rx + s) = 0$

This method is applicable if we can find out 2 numbers whose sum is equal to b and whose product is equal to ac.

Solve the equations by factorisation method.

Problem No.1. $\mathbf{x^2 - 5x + 6 = 0}$

$$x^2 - 3x - 2x + 6 = 0$$
$$x(x-3) - 2(x-3) = 0$$
$$(x-3)(x-2) = 0$$
$$x - 3 = 0 \qquad x - 2 = 0$$
$$\mathbf{x = 3} \qquad \mathbf{x = 2}$$

Problem No. 2. $\mathbf{x^2 - x - 12 = 0}$

$$x^2 - 4x + 3x - 12 = 0$$
$$x(x-4) + 3(x-4) = 0$$
$$(x+3)(x-2) = 0$$
$$x + 3 = 0 \qquad x - 4 = 0$$
$$\mathbf{x = -3} \qquad \mathbf{x = 4}$$

Problem No. 3. $\mathbf{x^2 - 3x - 10 = 0}$

$$x^2 - 5x + 2x - 10 = 0$$
$$x(x-5) + 2(x-5) = 0$$
$$(x-5)(x+2) = 0$$
$$x - 5 = 0 \qquad x + 2 = 0$$
$$\mathbf{x = 5} \qquad \mathbf{x = -2}$$

Problem No. 4. $\mathbf{x^2 - 6x + 8 = 0}$

$$x^2 - 4x - 2x + 8 = 0$$
$$x(x-4) - 2(x-4) = 0$$
$$(x-4)(x-2) = 0$$
$$x - 4 = 0 \qquad x - 2 = 0$$
$$\mathbf{x = 4} \qquad \mathbf{x = 2}$$

Problem No. 5. $\mathbf{9x^2 - 12x - 5 = 0}$

$$9x^2 - 15x + 3x - 5 = 0$$
$$3x(3x - 5) + 1(3x - 5)$$
$$(3x - 6)(3x + 1)$$
$$9x^2 + 3x - 15x - 5 = 0$$
$$3x(3x + 1) - 5(3x + 1) = 0$$
$$(3x + 1)(3x - 5) = 0$$
$$3x + 1 = 0 \qquad 3x - 5 = 0$$
$$3x = 1 \qquad 3x = 5$$
$$x = \frac{1}{3} \qquad x = \frac{5}{3}$$

Problem No. 6. $\mathbf{3x^2 - 14x + 11 = 0}$

$$3x^2 - 11x - 3x + 11 = 0$$
$$x(3x - 11) - 1(3x - 11)$$
$$(3x - 11)(x - 1)$$
$$3x - 11 = 0 \qquad x - 1 = 0$$
$$3x = 11 \qquad x = 1$$
$$x = \frac{11}{3}$$

2. By using Formula : The roots of quadratic equation $ax^2 + bx + c = 0$ and a is not equal to zero given by the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The 2 roots are $-b + \frac{\sqrt{b^2 - 4ac}}{2a}$

and

$b - \frac{\sqrt{b^2 - 4ac}}{2a}$

1. **Solve** $\mathbf{x^2 + 9x + 8 = 0}$ **by using the formula comparing** $\mathbf{x^2 + 9x + 8 = 0}$ **with** $\mathbf{ax^2 + bx + c = 0}$

   we get $a = 1 \quad b = 9 \quad c = 8$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-9 \pm \sqrt{9^2 - 4 \times 1 \times 8}}{2 \times 1}$$
$$x = \frac{-9 \pm \sqrt{81 - 32}}{2}$$
$$x = \frac{-9 \pm \sqrt{49}}{2}$$
$$x = \frac{-9 \pm 7}{2}$$

The two roots are $\frac{-2}{2}$, $\frac{-16}{2}$

$x = -1$, or $x = -8$

2. $3x^2 + 10x + 4 = 0$ by using formula.

a=3 b=10 c=4

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-10 \pm \sqrt{10^2 - 4 \times 3 \times 4}}{2 \times 3}$$

$$x = \frac{-10 \pm \sqrt{100 - 48}}{6}$$

$$x = \frac{-10 \pm \sqrt{52}}{6}$$

$$\frac{-10 \pm \sqrt{4 \times 13}}{6}$$

$$\frac{-10 \pm 2\sqrt{13}}{6}$$

$$\frac{-10 + 2\sqrt{13}}{6} \text{ or } \frac{-10 - 2\sqrt{13}}{6}$$

$x = -5 + 2\sqrt{13} = 2$ or $-5 - 2\sqrt{13}$

3. Solve the equation by using formula.

$2x^2 + 8x + 5 = 0$

a=2 b=8 c=5

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-8 \pm \sqrt{8^2 - 4 \times 2 \times 5}}{2 \times 2}$$

$$x = \frac{-8 \pm \sqrt{64 - 40}}{4}$$

$$x = \frac{-8 \pm \sqrt{24}}{4}$$

$$x = \frac{-8 \pm \sqrt{4 \times 6}}{4}$$

$$x = \frac{-8 + 2\sqrt{6}}{4} \text{ or } \frac{-8 - 2\sqrt{6}}{4}$$

$x = -2 + 2\sqrt{6}$ or $-2 - 2\sqrt{6}$